Robust decision making for magnetic fusion in the presence of model errors

Cosmin G. Petra*[1], Sanjay Mehrotra[2], Mihai Anitescu[1], and Emil Constantinescu[1]

[1]Mathematics and Computer Science Division, Argonne National Laboratory ({petra,anitescu,emconsta}@mcs.anl.gov)
[2]Industrial Engineering and Management Sciences Department, Northwestern University (mehrotra@iems.northwestern.edu)

Whole device design and operations for magnetic fusion presents multiple challenges. Modeling of the physical phenomena requires expensive models with fine details. Even when these are available and assembled, the amount of unresolved features remains significant and their effect on such devices with many critical contingencies needs to be assessed and mitigated. Moreover, the consequences of such contingencies requires a very fine performance/safety balance despite incomplete information.

A possible framework responsive to these aims is one of *robust risk-averse optimization* built on uncertainty quantification of complex physical processes, novel stochastic optimization techniques, and scalable algorithms for solving the resulting problems. Several challenges lie ahead of such an endeavor. The errors in the physical models and numerical methods need to be quantified in tractable ways. The resulting data assimilation problems must be efficiently solved on emerging architectures. The resulting stochastic optimization problem formulations needs to account for the unique risks attached to such devices, as well as for the missing information.

Model error The presence of model error is due to discretization, subscale effects as a result of filtering or model reduction among others. The model errors are usually represented as a collection of most often mutually independent identically distributed stochastic processes. More sophisticated approaches can represent the modeling error as multivariate normal processes constructed based on ad hoc functional kernels [7, 6]. While the former typically represents an inadequate approach, the latter approach is robust but can be suboptimal if the underlying physics is not considered [2]. In magnetic fusion energy sciences, most components have well defined physics, which needs to be taken into account in order to obtain an effective and tractable model error representation. For example this can be done by means of covariance functions whose structure is derived from simplified but physics-based models. Such approaches were defined in [3, 2] and used in conjunction with dynamic problems [4, 5]. The idea is to use simplified physics to build covariance models consistent with the mathematical-physics model. More effective propagation of model uncertainty can build on Bayesian approximation error ideas [8]. Moreover, the stochastic approaches need to take advantage of hierarchical representation of the whole device model components and models of different complexity (e.g, gyro-Landau-fluid model GLF and TGLF).

Numerical Data Assimilation Formulations that account for structural, parametric, and numerical errors need to be calibrated and fitted from data. Because the system physical state itself is now conceptually a random variable, it can present correlation with all the other quantities, a fact which results in complex correlation relationships. Solution to such problems has been provided by successive estimation, such as Kalman filtering, particle methods, and variational approaches, each with their

*corresponding author

challenges. Even when adapted to nonlinear approaches, the gain matrices of Kalman methods now need to be approximated, as they would not be storable. Particle methods require care since filter collapse is quite frequent for problems of this size. Variational methods need to contend with the sharply increased state space due to the lack of exact causality when model error is considered. In addition to computational intensity this also results in significant memory challenges. New low-memory strategies allow space-time computations at the cost of a few state vectors for the latter approach [1]. The optimal approach, however, is likely to involve combinations of ideas and required advanced from all these method classes.

Robust Optimization Robust risk-averse optimization is a tool that enables systematic risk mitigation and ensures extremely high safety requirements under multiple sources of uncertainty. The so-called gap control approach is one example of such optimization paradigm and consists of controlling the steady-state plasma shape under a electromagnetic field in tokamaks within a maximum gap of a few centimeters. This control problem also needs to control plasma instability specific to elongated plasma poloidal cross-sections in high performance tokamaks [14]. Furthermore, to lower the operating costs, the toroidal coils are designed to keep the maximum tolerable currents as low as possible, which results in limitations or constraints on the controls. Ensuring confined steady-state plasma flow, cost efficiency and robustness of stabilization under random disturbances are competing requirements that require a unified decision making framework: *robust risk-averse* optimization. This class of optimization problems are usually equipped with risk measures capable of robustly quantifying the systematic risk and driving the control process. One such risk measure is the conditional value-at-risk (CVAR) [12, 13], which offers a mechanism to quantify failures that might be encountered in the tail of distributions. It also enjoys properties favorable to computational tractability.

However, a *robust* decision making process needs to also account for model form and uncertainty quantification errors in addition to systematic risk. For example, in a robust control design framework, this is necessary in order for the controls to quantitatively avoid designs with considerable errors, sampling bias and/or high variance associated with model form errors. This essentially requires robustyfing risk-measures against model form and UQ errors. The resulting robust optimization model intimately depends on how the errors are characterized and several robust formulations can be obtained. In the simplest case, when a deterministic model for errors is available, for example in the form of an ambiguity set, the decision making problem results in a classic robust $\min\max$ problem. If a stochastic model is available for the model error, the (inner) robust decision making problem will take the form of a stochastic programming problem, where the minimization of the expectation with respect to model error, possibly plus a penalization of the variance, is seek. In the extreme case when the model errors cannot be fully specified by a deterministic ambiguity set or a distribution function, but by a set of distributions specified by a subset of moments or using a distance from a reference probability measure, the decision making process takes the form of a distributionally robust optimization problem. Computational tractability of such formulations can be reached based on a surface cutting algorithms such as [10] in the framework of semi-infinite optimization and with computational optimization techniques specialized to the stochastic structure of these problems, similar to [9, 11].

References

[1] M. Anitescu, X. Zeng, and E. Constantinescu. A low-memory approach for best-state estimation of hidden Markov models with model error. *SIAM Journal on Numerical Analysis (SINUM)*, 52(1):468–495, 2013.

[2] E. Constantinescu and M. Anitescu. Physics-based covariance models for Gaussian processes with multiple outputs. *International Journal for Uncertainty Quantification*, 3(1):47–71, 2013.

[3] E. Constantinescu, T. Chai, A. Sandu, and G. Carmichael. Autoregressive models of background errors for chemical data assimilation. *Journal of Geophysical Research*, 112:D12309, 2007.

[4] E. Constantinescu, A. Sandu, T. Chai, and G. Carmichael. Ensemble-based chemical data assimilation I: General approach. *Quarterly Journal of Royal Meteorological Society*, 133(626):1229–1243, July 2007.

[5] E. Constantinescu, A. Sandu, T. Chai, and G. Carmichael. Ensemble-based chemical data assimilation II: Covariance localization. *Quarterly Journal of Royal Meteorological Society*, 133(626):1245–1256, July 2007.

[6] D. Higdon. *Statistical Methods for Spatio-Temporal Systems*, chapter VI A primer on space-time modeling from a Bayesian perspective, pages 217–279. Chapman & Hall, 2007.

[7] D. Higdon, J. Gattiker, B. Williams, and M. Rightley. Computer model calibration using high-dimensional output. *Journal of the American Statistical Association*, 103(482):570–583, 2008.

[8] J. Kaipio and E. Somersalo. Statistical inverse problems: Discretization, model reduction and inverse crimes. *Journal of Computational and Applied Mathematics*, 198(2):493–504, 2007.

[9] M. Lubin, C. G. Petra, M. Anitescu, and V. Zavala. Scalable stochastic optimization of complex energy systems. In *Proceedings of 2011 International Conference for High Performance Computing, Networking, Storage and Analysis*, SC '11, pages 64:1–64:64, New York, USA, 2011. ACM.

[10] S. Mehrotra and D. Papp. A cutting surface algorithm for semi-infinite convex programming with an application to moment robust optimization. *SIAM Journal on Optimization*, 24(4):1670–1697, 2014.

[11] C. G. Petra, O. Schenk, M. Lubin, and K. Gärtner. An augmented incomplete factorization approach for computing the Schur complement in stochastic optimization. *SIAM Journal on Scientific Computing*, 36(2):C139–C162, 2014.

[12] R. T. Rockafellar and S. Uryasev. Optimization of conditional value-at-risk. *Journal of risk*, 2:21–42, 2000.

[13] R. T. Rockafellar and S. Uryasev. Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26(7):1443–1471, 2002.

[14] M. Walker, E. Schuster, D. Mazon, and D. Moreau. Open and emerging control problems in tokamak plasma control. In *47th IEEE Conference on Decision and Control, 2008*, pages 3125–3132, Dec 2008.