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A Multiphysics and Multiscale Coupling of Microturbulence with MHD Equilibria

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We will investigate the multiphysics and multiscale coupling between a gyrokinetic "microscopic" code for studying gyroradius-scale, turbulence-associated ion-acoustic and Alfven waves, and a "macroscopic" code for computing large-scale global equilibria based on the time-independent magnetohydrodynamic equations, in order to identify families of self-consistent global MHD equilibria that minimize the electrostatic potentials leading to microturbulence. By using the electromagnetic version [1] of the GTS code [2] for studying finite-β microturbulence and its nonlinear consequences under various MHD equilibrium conditions, and using the SPEC code [3] for calculating three-dimensional MHD equilibria with or without chaotic fields, we seek to confirm the predicted correlation [4] between the gyrokinetic evolution and the MHD equilibrium when the electrostatic potential vanishes.

The proposed work involves the scales ranging from the electron skin depth to the major radius of the machine, and includes the physics of both gyrokinetics and MHD. The work will be carried out by using these codes on the DoE high performance supercomputers, where excellent scaling of particle codes on these platforms with billions of particles using millions of processor cores has been demonstrated [5], and with a parallelized MHD equilibrium code [3].

The governing, gyrokinetic Vlasov-Maxwell system of equations can be derived from the Lagrangian $L = \frac{1}{2}mv^2 - q\phi + \frac{q}{c}\mathbf{v}\cdot\mathbf{A}$, using the usual gyrokinetic ordering and gyrophase-averaging [4]. A new treatment by calculating $A_\parallel$ and $\partial A_\parallel/\partial t$ has been devised, which has enabled us to study tearing physics [1] in GTS [3]. In the fluid limit, this set of equations can be reduced to pressure balance, vorticity equation and collisionless Ohm's law, respectively,

$$\mathbf{J}_\perp = \frac{c}{B}\mathbf{b}\times\nabla p, \tag{1}$$

$$(\frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla)\nabla_\perp^2\phi - 4\pi\frac{v_A^2}{c^2}\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0, \tag{2}$$

$$E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0, \tag{3}$$

with $\nabla^2\mathbf{A} = -(4\pi/c)\mathbf{J}$ and $\mathbf{B} = \nabla\times\mathbf{A}$, where v_A is the Alfven speed and $\mathbf{b}\equiv\mathbf{B}/B$. When $\phi\rightarrow 0$, which gives $\partial A_\parallel/\partial t \rightarrow 0$ from Eq.(3), then from Eq.(2) we obtain $\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0$. Thus, the gyrokinetic equations exhibit the quasi-neutral, static property in the absence of the fluctuating potential. It is this connection between the time-dependent, micro system to the time-independent, macro system that forms the basis of our approach. Equations (1)-(3) are similar to those in Ref. [6], except for the $\mathbf{J}_\perp$ term in Eq. (2), where, in addition to a magnetic drift current, a divergence free diamagnetic current has been added in the present formulation.

To obtain a tractable, theoretical and numerical model of MHD equilibria, simplifying assumptions must be made, e.g. that transport along the magnetic field greatly exceeds that perpendicular to the field, so that $\mathbf{B} \cdot \nabla p = 0$. MHD equilibria are defined as those magnetic configurations that extremize the energy functional, defined as a volume integral over the entire plasma domain of

$$W \equiv \int_{\mathcal{V}} \left(\frac{p}{\gamma - 1} + \frac{B^2}{8\pi} \right) dv, \tag{4}$$

from which we can see that equilibrium codes treat the physics in the length scale opposite to that of gyrokinetics. To obtain non-trivial solutions, appropriate topological constraints on the magnetic field must be enforced. The SPEC code [3] parallelizes the volume integral, $\int_{\mathcal{V}} dv \equiv \sum_{i=1}^{N} \int_{\mathcal{V}_i} dv$. In each sub-volume, $\mathcal{V}_i$, the variation of the energy functional is performed under the constraints of conserved toroidal and poloidal fluxes, and helicity, $K_i \equiv \int_{\mathcal{V}_i} \mathbf{A} \cdot \mathbf{B}\, dv$. States that extremize this *multi-region* energy principle satisfy $\mathbf{j} = \mu_i \mathbf{B}$ in each of the $\mathcal{V}_i$, where μ_i is a Lagrange multiplier introduced in each volume to enforce the helicity constraint. In each $\mathcal{V}_i$, the magnetic field may "relax" and islands and chaotic fields may emerge, and across the "ideal" interfaces that separate the sub-volumes the total pressure must be constant, $[[p + B^2/8\pi]] = 0$. These equations reduce to $\nabla p = \mathbf{J} \times \mathbf{B}/c$ as the number of volumes, N, increases.

By passing the global parameters, such as pressure, current and magnetic field, between the two codes, we plan to discover magnetic configurations which would give minimum transport and minimum chaos, and thus improved confinement in tokamaks. This novel approach, which has not been attempted before, is different from the conventional "integrated-modelling" approach of, for example, M3D-C_1 and NIMROD, which seek to incorporate both the transport and the equilibrium physics within the same numerical architecture; but, because of the extreme disparity in the length scales between the gyrokinetics and the equilibrium, and the overwhelming computational burden that would otherwise result, these codes must employ simplified transport physics, and therefore may not address the issue of gyrokinetic turbulence adequately. Our approach is iteratively to "decouple" the transport problem from the equilibrium problem, so that each may be treated accurately, and, then "couple" them through parameter exchanges.

[1] E. A. Startsev, W. X. Wang and W. W. Lee, "Gyrokinetic simulation of the collisional micro-tearing mode instability", International Sherwood Theory Conference, New York (2015).

[2] W. X. Wang, Z. Lin, W. Tang, W. W. Lee, S. Ethier, J. L. V. Lewandowski *et al.*, Phys. Plasmas **13**, 092525 (2006).

[3] S. R. Hudson, R. L. Dewar, G. Dennis, J. Hole, M. McGann, G. von Nessi *et al.*, Phys. Plasmas **19**, 112502 (2012).

[4] W. W. Lee "Issues Related to Finite-β Gyrokinetics: 1) MHD and Equilibrium, and 2) Singularly Perturbed Equation", International Sherwood Theory Conference, New York (2015).

[5] S. Ethier. C. S. Chang, S. Ku, W. W. Lee, Z. Lin, W. M. Tang and W. X. Wang, "NERSC impact on advances of global gyrokinetic PIC codes for fusion energy research", Special Issue for the 40th Anniversary of NERSC (2015).

[6] W. W. Lee and H. Qin, Phys. Plasmas **10**, 3196 (2003).