Parallelization and further development of stability code MARS

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Topic A: Disruption prevention, avoidance, and mitigation. Oral presentation is not required.

There are a few available linear ideal MHD stability codes designed for stability analysis of toroidal axisymmetric equilibria, like ERATO [1], PEST [2], GATO [3], DCON [4], KINX [5], NOVA [6], etc. There are also more advanced MHD stability codes like MARS [7]-[14], AEGIS [15],[16], MISK [17], etc., in which nonideal effects, such as plasma resistivity or particles kinetic effects, are included. All these codes are very well tested and reliable. However they are single processor codes, some of them written more than 20 years ago.

Due to computing power and memory size constraints, single processor codes have limited scope of applicability. They can handle rather limited numbers of radial and poloidal grid points (poloidal harmonics) and are usually limited to a calculation of modes with low toroidal mode numbers n. This significantly limits the scope of physics problems that single processor stability codes can solve.

FAR-TECH, Inc. is working on parallelization of the MARS code as a Small Business Innovative Research project. Parallelization of MARS will extend its capabilities, and will significantly improve its performance.

MARS calculates eigenmodes in 2D axisymmetric toroidal equilibria within the ideal MHD, resistive MHD, and MHD-kinetic plasma models. In addition, a non-uniform plasma rotation model and a model for external coils and magnetic feedback characteristics are included in MARS. The code is a powerful tool for studying MHD and MHD-kinetic instabilities and it is widely used by the fusion community. This makes MARS the most suitable choice for parallelization.

The initial version of the code was developed in the early 1990s by A. Bondeson, G. Vlad and H. Lütjens [7]-[9] with the goal to study nonideal plasma instabilities. This version of the code solved linearized resistive MHD equations in axisymmetric toroidal equilibrium.

Later the code was extended by Chu *et al* [10] with added non-uniform plasma rotation model and viscous damping models. The viscous damping models included a parallel sound wave damping model [18], a kinetic neoclassical damping model [19], and an anomalous perpendicular viscosity model. MARS was further extended to include external coils and various feedback characteristics by Liu *et al* [11] and to add more physics-based damping models, including the mode resonance with bounce motions of the thermal ions in the plasma [12].

The last extension of the code was made in 2008 [13] to include the kinetic effect of the diamagnetic drifts of the particles for the study of the RWM in the low rotation frequency regime. It solves a self-consistent nonlinear eigenvalue problem which couples linear MHD equations with perturbed kinetic pressure tensors, analytically derived by solving the linearized drift kinetic equation. The kinetic pressure tensor terms are represented in the form of kinetic integrals over the particle velocity phase space, assuming vanishing banana width for the particle orbit.

The plasma equations in MARS correspond to single fluid MHD model with toroidal flow and kinetic contributions [13],

$$(\gamma + in\Omega)\,\boldsymbol{\xi} = \boldsymbol{v} + (\boldsymbol{\xi}\cdot\boldsymbol{\nabla}\Omega)\,R^2\boldsymbol{\nabla}\phi\,,$$

$$\rho\left(\gamma+in\Omega\right)\boldsymbol{v}=-\boldsymbol{\nabla}\cdot\boldsymbol{p}+\boldsymbol{j}\times\boldsymbol{B}+\boldsymbol{J}\times\boldsymbol{Q}-\rho\left[2\Omega\hat{\boldsymbol{Z}}\times\boldsymbol{v}+\left(\boldsymbol{v}\cdot\boldsymbol{\nabla}\Omega\right)R^2\boldsymbol{\nabla}\phi\right],$$
$$\left(\gamma+in\Omega\right)\boldsymbol{Q}=\boldsymbol{\nabla}\times\left(\boldsymbol{v}\times\boldsymbol{B}\right)-\boldsymbol{\nabla}\times\left(\eta\boldsymbol{j}\right)+\left(\boldsymbol{Q}\cdot\boldsymbol{\nabla}\Omega\right)R^2\boldsymbol{\nabla}\phi,$$
$$\left(\gamma+in\Omega\right)p=-\boldsymbol{v}\cdot\boldsymbol{\nabla}P,$$
$$\boldsymbol{j}=\boldsymbol{\nabla}\times\boldsymbol{Q},$$
$$\boldsymbol{p}=p\boldsymbol{I}+p_{\parallel}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}+p_{\perp}\left(\boldsymbol{I}-\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\right),$$

where the variables $\boldsymbol{\xi}$, $\boldsymbol{v}$, $\boldsymbol{Q}$, $\boldsymbol{j}$, $\boldsymbol{p}$ are the plasma displacement, perturbed velocity, magnetic field, current, and pressure tensor, respectively. ρ is the unperturbed plasma density. The linear problem is formulated as an eigenvalue problem, with γ being the eigenvalue, which is corrected by a Doppler shift $in\Omega$, with n being the toroidal mode number, and Ω the plasma rotation frequency along the toroidal angle ϕ. The equilibrium field, current, and pressure are denoted by $\boldsymbol{B}$, $\boldsymbol{J}$, and P, respectively. R is the plasma major radius, $\hat{\boldsymbol{Z}}$ is the unit vector in the vertical direction, and $\boldsymbol{I}$ is the unit tensor.

The kinetic terms enter into the MHD equations via the perturbed kinetic pressure tensors, where p is the scalar fluid pressure perturbation, $p_{\parallel}(\boldsymbol{\xi}_{\perp})$ and $p_{\perp}(\boldsymbol{\xi}_{\perp})$ are the parallel and perpendicular components of the kinetic pressure perturbations, respectively. The full pressure tensor $\boldsymbol{p}$ is self-consistently included into the MHD formulation via the momentum equation.

The perturbed kinetic pressure tensors are calculated [20],[21] in terms of the perturbed particles distribution function f_L^1 in the Lagrangian frame,

$$p_{\parallel}e^{-i\omega t+in\phi}=\sum_{\text{e,i}}\int d\Gamma M v_{\parallel}^2 f_L^1\,,\quad p_{\perp}e^{-i\omega t+in\phi}=\sum_{\text{e,i}}\int d\Gamma M v_{\perp}^2 f_L^1\,,$$

with $exp(-i\omega t+in\phi)$ dependence explicitly assumed for the perturbation, and $\omega=i\gamma$. The integral is carried out over the particle velocity space Γ. M is the particle mass, $v_{\parallel}$ and $v_{\perp}$ are the parallel and perpendicular (to the equilibrium magnetic field) velocities of particle bounce motion. f_L^1 is time-dependent solution of linearized drift-kinetic equation which corresponds to plasma response on the fields in the mode with specified time and ϕ dependence. Multiplying the above equations by $\exp(i\omega t)$ and integrating over time gives the amplitudes of the pressure tensor components.

In MARS, the effect of finite radial excursion width of particles across the magnetic surfaces is neglected (zero banana width approximation). As a result, f_L^1 and $p_{\parallel}$, $p_{\perp}$ on a given magnetic surface linearly depend on $\boldsymbol{\xi}_{\perp}$ and $Q_{\parallel}$ defined on the same magnetic surface. Since response of f_L^1 on frequency of perturbation is rather complicated, the dependence of the pressure tensor amplitudes on the eigenvalue γ is also complicated and nonlinear.

The eigenvalues γ are found by the inverse vector iteration [1],[22]. MARS parallelization approach includes: 1) parallelization of the construction of the matrix for the eigenvalue problem and 2) parallelization of the solution of the formulated eigenvalue problem. Parallelization of the matrix construction is performed by distributing the calculations between processors assigned to different magnetic surfaces. Parallelization of the eigenvalue solver is performed by repeating present MARS algorithm using ScaLAPACK library and parallel procedures.

We propose to further extend the scope of the MARS code by calculating exact particles trajectories in the kinetic contributions without the zero banana width approximation. Such calculation is feasible on modern parallel clusters and it will allow to accurately predict stability of plasma, containing NBI fast particles and fusion alphas.

References

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