Resistive DCON and Beyond

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Rapid progress has recently been made on extending the ideal MHD DCON stability code to treat resistive instabilities and similar modes characterized by a narrow singular layer about mode rational surfaces. These are among the most important modes in determining stability and disruption limits on ITER operation. The new package of codes provides a fast, accurate, and robust determination of the linear and nonlinear evolution of these modes in realistic equilibrium configurations. New work will extend the physics content of the resistive layers, using recently-developed numerical methods, to include complete fluid, drift kinetic, and nonlinear treatments.

DCON is widely used for fast, accurate determination of ideal MHD stability of axisymmetric toroidal plasmas, including ideal and resistive interchange modes, ballooning modes with large toroidal mode number n, and low-n fixed and free boundary modes. It is thoroughly verified against other stability codes and validated against experimental observations. Running on one core of a modern workstation, for a single-null NSTX case with $\beta_N = 5.6$, $q_a = 11.8$, and aspect ratio 1.5 runs in about 4 seconds for toroidal mode number $n = 1$ and 10 seconds for $n = 2$. A companion white paper has been submitted on parallelization of ideal DCON for real-time feedback control of ITER profile evolution.

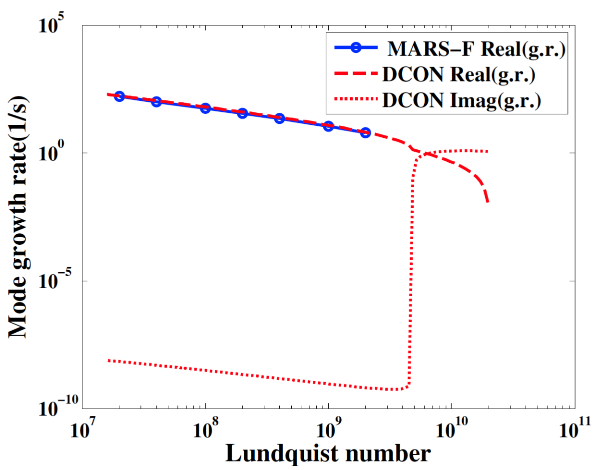
To deterrmine the stability to low-n internal and free-boundary ideal modes, DCON uses adaptive integrator LSODE to integrate the 2D Newcomb equation, the Euler-Lagrange equation for minimizing the perturbed energy δW, from the magnetic axis to the plasma-vacuum interface. This is a high-order coupled system of complex ODEs with independent variable ψ, the normalized poloidal flux, and independent variable Ξ, a vector of Fourier components of the normal perturbed displacement. Mathematically this is an initial value problem, with integration proceeding in only one direction, outward. Boundary conditions at each mode-rational surface, where $m = nq$ for one of the poloridal Foruier components, exclude the large resonant component, relaunch a new small resonant component, and impose C^1 continuity on the nonresonant components.

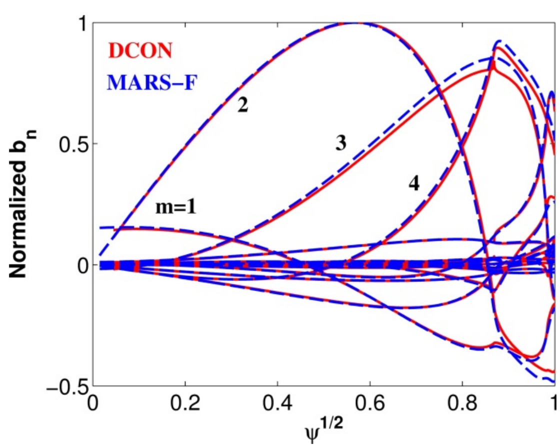
For large Lundquist number $S = \tau_R/\tau_A >> 1$, characteristic of thermonuclear plasmas, there is are narrow singular layers about each mode-rational surface where finite resistivity, inertia, and other non-ideal effects are important. The most accurate and efficient way to treat these modes is by matched asymptotic expansions. The computational domain is partitioned into multiple "outer" ideal regions and "inner" resistive regions, which are then coupled by matching the large and small resonant solutions between inner and outer regions to obtain global growth rates and eigenfunctions. DCON is used to determine the outer region matching data, a matrix generalization of the scalar quantity commonly called Δ'. A new code DELTAC is used to compute the corresponding inner region matching data. Inner and outer matching data are combined in the MATCH code into a matching matrix $\mathbf{M}$, a function of the complex growth rate s. The dispersion relation det $\mathbf{M}(s) = 0$ is solved for multiple roots s, both unstable and stable. For each root, MATCH computes and visualizes the global eigenfunction. For verification, the growth rate and eigenfunction are compared with the straight-through MARS code solving the same physical equations.

Straightforward extension of adaptive integration to treat the outer region matching data converts the initial value problem, well-suited to ideal stability calculations, into a numerically unstable and illposed shooting problem. Once this was recognized, the shooting method was replaced by a singular Galerkin method, expanding the unknowns in basis functions on a grid, converting the problem to Cholesky factorization solution of a banded complex matrix, while reusing most of the existing infrastructure of DCON. While this approach was previously used in the PEST III code, the implementation left room for large improvements, primarily in the choice of Galerkin basis functions. Linear finite elements are replaced with C^1-continuous Hermite cubics, well-suited to resolve nonresonant components. Additional basis functions derived from the high-order Frobenius expansion about the singular surface, faciliated by the bicubic spline representation of equilibrium quantities, greatly improve resolution of the resonant components and permit operation in regions of low shear and high β, where the Mercier index $\mu > 1$. An improved algorithm for packing the grid near the singular surfaces, allows user specification of the ratio of the largest to smallest grid cell. These improvements lead to much more rapid and robust convergence than the PEST III code.

Several methods of numerical solution of the resistive inner region equations of Glasser, Greene, and Johnson have been developed. All of these codes are found to fail for the parameters of the MARS benchmark cases we have studies, because these parameters stress the numerical methods much more than the published test cases. We have written a new code DELTAC to solve the inner region equations with another Galerkin expansion using Hermite cubics.

These figures show a comparison of DCON and MARS results for an equilibrium with $n = 1$ and two singular surfaces at $q = 2$ and 3. The figure on the left shows the complex growth rate vs. Lundquist number S. The figure on the right shows a comparison of Fourier components of the eigenfunction vs. scaled radius. The agreement is excellent, and DCON is about 100 times faster and functions reliably at values of S greater than the limit of the MARS code.





Future plans beyond resistive DCON are to add more complete physics to the resistive inner region model, including equilibrium toroidal rotation, anisotropic viscosity and thermal conduction, Hall and electron pressure effects, and finite Larmor radius and banana width. This can all be done with the newly developed numerical methods, using Braginskii closures for short mean free path and drift kinetic closures for long mean free path. Beyond that, we will develop a 2D nonlinear treatment of the helically symmetric inner regions, with coupling of multiple singular layers through linear outer regions. The advantage of implementing these improvements only in the inner region is a large reduction in the size and dimensionality of the computational domain, resulting in greatly improved speed and accuracy.